KNT/KW/16/5168

Bachelor of Science (B.Sc.) Semester—V (C.B.S.) Examination

MATHEMATICS

Paper—1

(M_9-Analysis)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the Fourier series for the function f(x) defined as :

$$f(x) = x + \frac{\pi}{2} \quad , \quad -\pi < x \le 0$$
$$= \frac{\pi}{2} - x \quad , \quad 0 < x < \pi$$

and deduce that at x = 0

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.$$ 6

(B) Express f(x) = sin x as a Fourier cosine series in $0 \le x < \pi$. 6

OR

(C) Find the Fourier series expansion for the function

$f(x) = x - x^2, \ -1 \le x \le 1.$ 6

(D) Find the Fourier cosine series for the function f(x) defined by :

$f(x) = 2 \ , \ 0 \le x \le 1$

$= 0 \ , \ 1 < x < 2$ 6

UNIT—II

2. (A) Prove that if function f is continuous on [a, b], then $f \in R(\alpha)$ on [a, b]. 6

(B) If $f \in R(\alpha)$ on [a, b] and if $a < c < b$, then prove that $f \in R(\alpha)$ on [a, c] and $f \in R(\alpha)$ on [c, b]. Also $\int_a^c f\, d\alpha + \int_c^b f\, d\alpha = \int_a^b f\, d\alpha$. 6

OR

(C) Explain when the function f is Riemann-Steiltjes (R-S) integrable on [a, b]. Hence show that a constant function f(x) = k is R-S integrable on [a, b]. 6

(D) If $f \in R(\alpha)$ on [a, b] and if there is a differentiable function on [a, b] such that $F' = f$, then prove $\int_a^b f(x)\, dx = F(b) - F(a)$. 6

UNIT—III

3. (A) If w = f(z) = u(x, y) + iv(x, y) is an analytic function at any point z = x + iy of its domain D, then show that in polar form the Cauchy-Riemann equations are :

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$ 6

(B) (i) If $f(z) = u(r, \theta) + iv(r, \theta)$ is an analytic function is z-plane, then prove that

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0.$$

(ii) Find p such that the function $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic. 6

OR

(C) Prove that the function $u = x^3 - 3xy^2$ satisfies Laplace equation and determine corresponding analytic function in a finite region D. 6

(D) If f(z) = u + iv is analytic function of z, then prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(z)|^2.$$ 6

UNIT—IV

4. (A) Let the region R in z-plane be bounded by lines x = 0, y = 0, x = 2, y = 1. Find the region R′ in w-plane by using the transformation $w = \sqrt{2}\, e^{i\pi/4} . z + (1 - 2i)$. 6

(B) Show that every general bilinear transformation

$$w = \frac{az+b}{cz+d} \text{ is the resultant of}$$

$$w = z + d,\ w = \frac{1}{z} \text{ and } w = \beta z,\ c \neq 0.$$ 6

OR

(C) Show that if there are two distinct invariant points p and q then the normal form of bilinear transformation $w = \frac{az+b}{cz+d}$ where $ad - bc \neq 0$ is

$$\frac{w-p}{w-q} = K\left(\frac{z-p}{z-q}\right).$$ 6

(D) Show that the transformation $w = \frac{2z+3}{z-4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$. 6

UNIT—V

5. (A) If $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$, $-\pi \leq x \leq \pi$ then show that $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$.

1½

(B) If the function f(x) is defined as :

$$f(x) = \begin{cases} \pi & , \quad -\pi \leq x \leq \frac{\pi}{2} \\ 0 & , \quad \frac{\pi}{2} < x \leq \pi \end{cases}$$

then find the value of f(x) at the point of discontinuity by Dirichlet's condition. 1½

(C) If $f_1(x) \le f_2(x)$ on $[a, b]$ then prove that $\int_a^b f_1 d\alpha \le \int_a^b f_2 d\alpha$. 1½

(D) Prove that if $f \in R(\alpha)$ and $g \in R(\alpha)$, then $f \cdot g \in R(\alpha)$. 1½

(E) If $u = x^3 - 3xy^2$, then find its harmonic conjugate v. 1½

(F) Prove that $f(z) = xy + iy$ is not analytic. 1½

(G) Find the fixed points of the bilinear transformation $w = \frac{3iz + 1}{z + i}$. 1½

(H) Find whether $w = \frac{2z + 1}{4z + 2}$ is a bilinear transformation. 1½